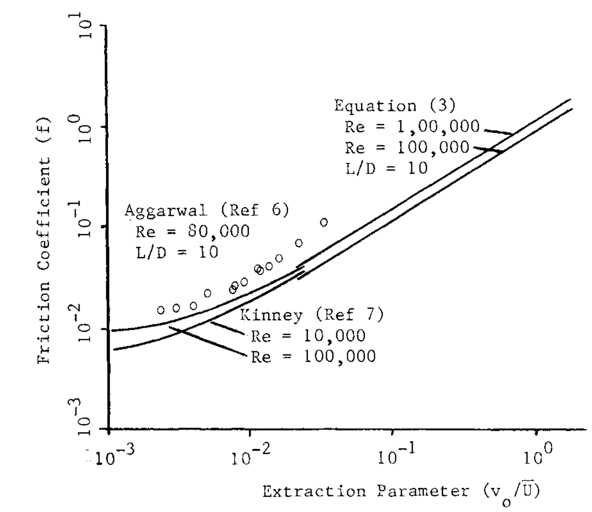

Fig. 3 Comparison with other models.

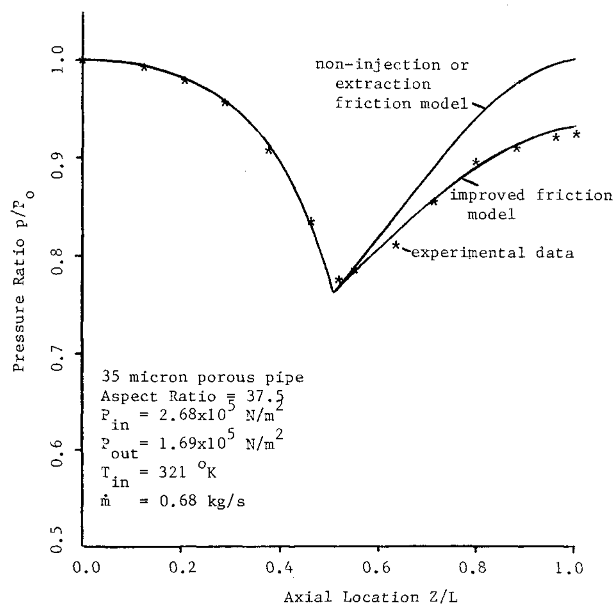

Fig. 4 1-D model with improved friction model.

1,000,000 and for extraction parameters between 0.015 and 2.0. This represents both larger extraction parameters and axial Reynolds numbers than studied by Aggarwal or Kinney.

To test the new friction models, a simplified flow model was studied that assumed the flow was compressible, one-dimensional, adiabatic, and steady. The study conducted compared two friction models. One model assumed that the friction coefficient was the same as used for nonblowing smooth pipe calculations.

$$f(\text{laminar}) = \frac{16}{Re}, \qquad f(\text{turbulent}) = \frac{0.046}{Re^{1/5}} \tag{5}$$

A smooth transition from the laminar region to the turbulent region was used.[3]

The other friction coefficient model used the expressions developed earlier in this paper [Eqs. (2) and (3)]. Figure 4 compares the numerical results from the two friction models with the experimental data. Extremely good results were obtained with the improved friction model.

Acknowledgment

The numerical portion of this work was supported by Dr. Joe Shang of the U.S. Air Force Flight Dynamics Laboratory, Dayton, Ohio, who wrote the basic computer code modified to solve this problem.

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Effects of Wetted Wall on Laminar Mixed Convection in a Vertical Channel

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Introduction

THE effects of mass diffusion on natural convection heat transfer have been studied widely for external[1,2] and internal[3] flows. As far as pure forced convection flows are concerned, combined heat and mass transfer in laminar forced convection flow over a flat surface was studied by Chow and Chung.[4] For mixed convection, Santarelli and Foraboschi[5] investigated the influence of natural convection on laminar forced convection in a reacting fluid.

Despite the fact that the influence of combined buoyancy forces of heat and mass diffusion was shown to be relatively important,[1-5] it has not received enough attention. In this Note, consideration is given to the effects of coupled thermal and mass diffusion on laminar forced convection. Particular attention is given to investigating the role of latent heat transfer, in association with the liquid film evaporation from the wetted channel wall, in laminar mixed convection heat transfer.

Analysis

The geometry to be examined is a vertical parallel-plate channel with width $2b$. The channel wall is wetted by thin water films. The loss of evaporated water in the film is compensated by the injection of additional water through the

Received Sept. 9, 1987; revision received Jan. 8, 1988.

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porous wall. With good control of the water injection and the appropriate choice of the porosity of porous channel wall, the film on the inside surface of the channel can be maintained extremely thin so that it can be regarded as stationary and at the same uniform temperature as the channel wall temperature T_w, which is higher than the ambient temperature T_0. By introducing Boussinesq's approximation, laminar mixed convection flow of the moist air in a vertical channel resulting from the combined buoyancy effects of thermal and mass diffusion, with the boundary-layer approximation adopted, can be described in dimensionless form by the following basic equations as

Continuity:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

Axial momentum:

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\mathrm{d}P}{\mathrm{d}X}+\frac{4(Gr_T\theta+Gr_MW)}{Re}+\frac{\partial^2 U}{\partial Y^2} \tag{2}$$

Energy:

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial Y^2}+\frac{A}{Sc}\frac{\partial\theta}{\partial Y}\frac{\partial W}{\partial Y} \tag{3}$$

Species diffusion for water vapor:

$$U\frac{\partial W}{\partial X}+V\frac{\partial W}{\partial Y}=\frac{1}{Sc}\frac{\partial^2 W}{\partial Y^2} \tag{4}$$

In nondimensionalizing the governing equations, the following dimensionless variables were introduced:

$$\begin{aligned}
X &= 4x/(b\cdot Re) & V &= vb/\nu\\
U &= u/u_0 & W &= (w-w_0)/(w_w-w_0)\\
\theta &= (T-T_0)/(T_w-T) & Re &= u_0(4b)/\nu\\
P &= (p-p_0)/(\rho u_0^2) & Gr_M &= g(M_a/M_v-1)\\
Gr_T &= g(T_w-T_0)b^3/(T_0\nu^2) & &\quad\times(w_w-w_0)b^3/\nu^2\\
A &= [(c_{pv}-c_{pa})/c_p](w_w-w_0) & &\\
Y &= y/b & &
\end{aligned} \tag{5}$$

Here x and y are the vertical and horizontal coordinates with the corresponding velocities u and v, respectively, and T, p, and w are the mixture temperature, pressure, and mass fraction of water vapor. Subscripts 0 and w denote the conditions at the inlet and wall, while the properties for the dry air and water vapor are distinguished by the subscripts a and v. ν, c_p, and M are the viscosity, specific heat, and molecular weight, respectively. Re, Pr, and Sc are the Reynolds, Prandtl, and Schmidt numbers, and Gr_T and Gr_M are the heat- and mass-transfer Grashof numbers. The interfacial mass fraction of water vapor on the wetted wall is[3]

$$w_w=p_wM_v/[p_wM_v+(p-p_w)M_a] \tag{6}$$

in which p_w is the partial pressure of water vapor at the interface.

In this Note, the thermophysical properties of the mixture are taken to be constant and are evaluated by the one-third rule. This special method of computing the properties is found to be appropriate for the study of combined heat- and mass-transfer problems.[4] Complete details on the evaluation of properties of air, water vapor, and their mixture are available in Ref. 6.

The governing equations are subject to the following boundary conditions:

$$X=0,\quad U=1,\quad \theta=0,\quad W=0,\quad P=0,$$

$$Y=0,\quad \frac{\partial U}{\partial Y}=0,\quad \frac{\partial\theta}{\partial Y}=0,\quad \frac{\partial W}{\partial Y}=0$$

$$Y=1,\quad U=0,\quad V=V_W,\quad \theta=1,\quad W=1 \tag{7}$$

in which

$$V_w=-\frac{w_w-w_0}{1-w_w}\frac{1}{Sc}\frac{\partial W}{\partial Y}\bigg|_{Y=1} \tag{8}$$

The energy transport from the wetted wall to the moist air in the presence of mass transfer depends on two related factors: the fluid temperature gradient at the wetted wall and the rate of mass transfer.[7] The total heat transfer from the wetted wall can then be expressed as

$$q_x''=q_s''+q_\ell''=k\frac{\partial T}{\partial y}\bigg|_{y=b}+\frac{\rho Dh_{fg}}{1-w_w}\frac{\partial w}{\partial y}\bigg|_{y=b} \tag{9}$$

The local Nusselt number defined by

$$Nu_x=h(4b)/k=q_x''(4b)/[k(T_w-T_b)] \tag{10}$$

can be written as

$$Nu_x=Nu_s+Nu_\ell \tag{11}$$

where

$$Nu_s=\frac{4}{1-\theta_b}\frac{\partial\theta}{\partial Y}\bigg|_{Y=1} \tag{12}$$

$$Nu_\ell=\frac{4S}{(1-\theta_b)(1-w_w)}\frac{\partial W}{\partial Y}\bigg|_{Y=1} \tag{13}$$

Here S signifies the relative importance of energy transport through species diffusion to that through thermal diffusion

$$S=\rho Dh_{fg}(w_w-w_0)/[k(T_w-T_0)] \tag{14}$$

in which D is the mass diffusivity and h_{fg} the latent heat of evaporation.

Results and Discussion

To obtain enhanced accuracy in the numerical computation, grids are chosen to be uniform in the y direction but nonuniform in the x direction to account for the drastic variations of velocity, temperature, and concentration in the near-entrance region ($\Delta x_1=10^{-4}$, $\Delta x_i=1.06\times\Delta x_{i-1}$, $i=2,3,\ldots$). Several arrangements of grid points in the x and y directions are tested. It is found that the differences in the local Nusselt number for the computations using 101×61 and 201×121 grids are always less than 1%. Accordingly, the 101×61 grid is sufficient in the computations.

In this Note, calculations are especially performed for the moist air flowing in a vertical channel with a channel width of 1.5 cm.[3] All of the cases selected in the computations are listed in Table 1.

To demonstrate the relative effects of heat transfer through sensible heat exchange and latent heat exchange, both Nu_s and

Table 1 Values of major parameters for various cases (T in degrees Centigrade; ϕ in percent)

Case	Re	T_0	T_w	ϕ	Pr	Sc	Gr_T	Gr_M	S
I	2000	20	21	50	0.71	0.60	62.4	91.3	23.3
II	2000	20	21	80	0.71	0.60	62.4	42.8	10.9
III	2000	20	40	50	0.70	0.59	1120.9	393.4	5.4
IV	2000	20	40	80	0.70	0.59	1120.9	349.9	4.8
V	2000	20	60	50	0.70	0.59	2016.5	1123.3	8.3
VI	400	20	40	50	0.70	0.59	1120.9	393.4	5.4

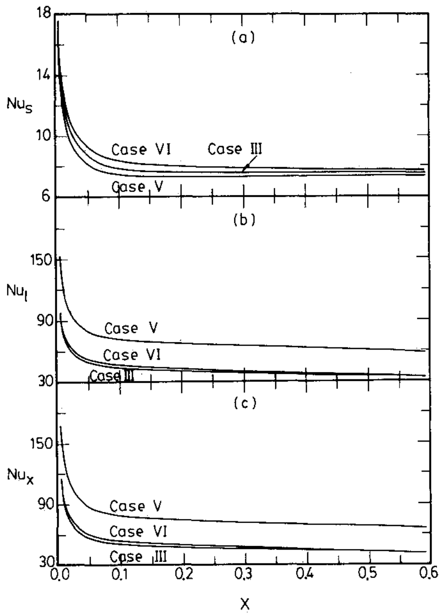

Fig. 1 Local Nusselt number for a) sensible heat, b) latent heat, c) overall along the channel. Case III: $Re = 2000$, $T_0 = 20°C$, $T_w = 40°C$, $\phi = 50\%$; case V: $T_w = 60°C$; case VI: $Re = 400$, $T_w = 40°C$.

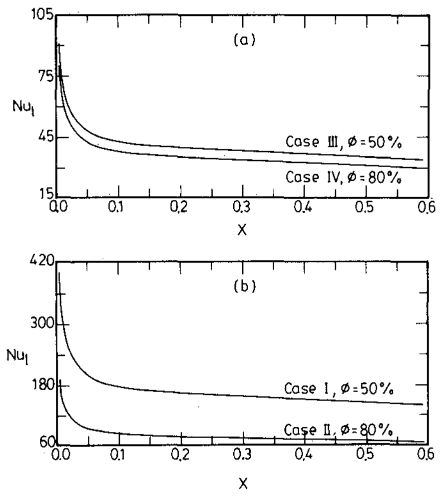

Fig. 2 Local Nusselt number (latent heat): a) $T_w = 40°C$ and b) $T_w = 21°C$ at different ambient humidity with $Re = 2000$.

Nu_ℓ are presented in Fig. 1. Inspection of the Nu_s variations with X reveals that Nu_s is slightly larger for flow with smaller Re (cases III and VI). This is a direct consequence of the larger buoyancy effect (higher Gr_T/Re and Gr_M/Re). The effect of wetted wall temperature on the sensible heat transfer is also shown in Fig. 1a. It is clear that Nu_s is smaller for a higher wetted wall temperature (cases III and V). This is in line with the findings by Carter and Gill[8] that a larger water vapor evaporation associated with a higher T_w causes the temperature profiles to flatten, which then results in less sensible heat transfer.[9]

An opposite trend is observed for Nu_ℓ in Fig. 1b, i.e., the flow with $T_w = 60°C$ shows a higher value for Nu_ℓ (cases III and V). This is simply because latent heat trasfer is determined mainly by the parameter S in Eq. (13), which is larger for a higher T_w, as is evident from Table 1. It becomes apparent by comparing the magnitudes of Nu_s and Nu_ℓ that heat transfer due to latent heat exchange is much more effective.

It is interesting to examine the effect of the ambient relative humidity ϕ on the transport of the latent heat. Depicted in Fig. 2 is the local Nusselt number Nu_ℓ. Close examination of these two figures reveals that an increase in ϕ results in a slight decrease in Nu_ℓ when $(T_w - T_0)$ is large, while the decrease in Nu_ℓ is rather substantial when $(T_w - T_0)$ is very small. This is because the extent of mass transfer becomes more significant when the thermal driving force $(T_w - T_0)$ is smaller. Hence, a slight increase in ϕ leads to a pronounced decrease in latent heat exchange for the system with a small $(T_w - T_0)$.

It is also noticed that the axial distributions of Sherwood number Sh resemble those of Nu_s.

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